IMPEDANCE MEASUREMENTS AT HIGH FREQUENCIES WITH STANDARD PARTS

INTRODUCTION

● THE ADVENT OF COMMERCIAL TELEVISION and a general increase in interest in high-frequency transmission have greatly stimulated the contemporary demand for precise measurements of impedance at high and ultra-high frequencies. During the past year, the General Radio Company, to meet the need for improved circuit elements, has announced several new units especially designed for high-frequency use. Together with instruments that have been available for a longer period of time, these new units round out the following complete line of components for use in impedance measurements:

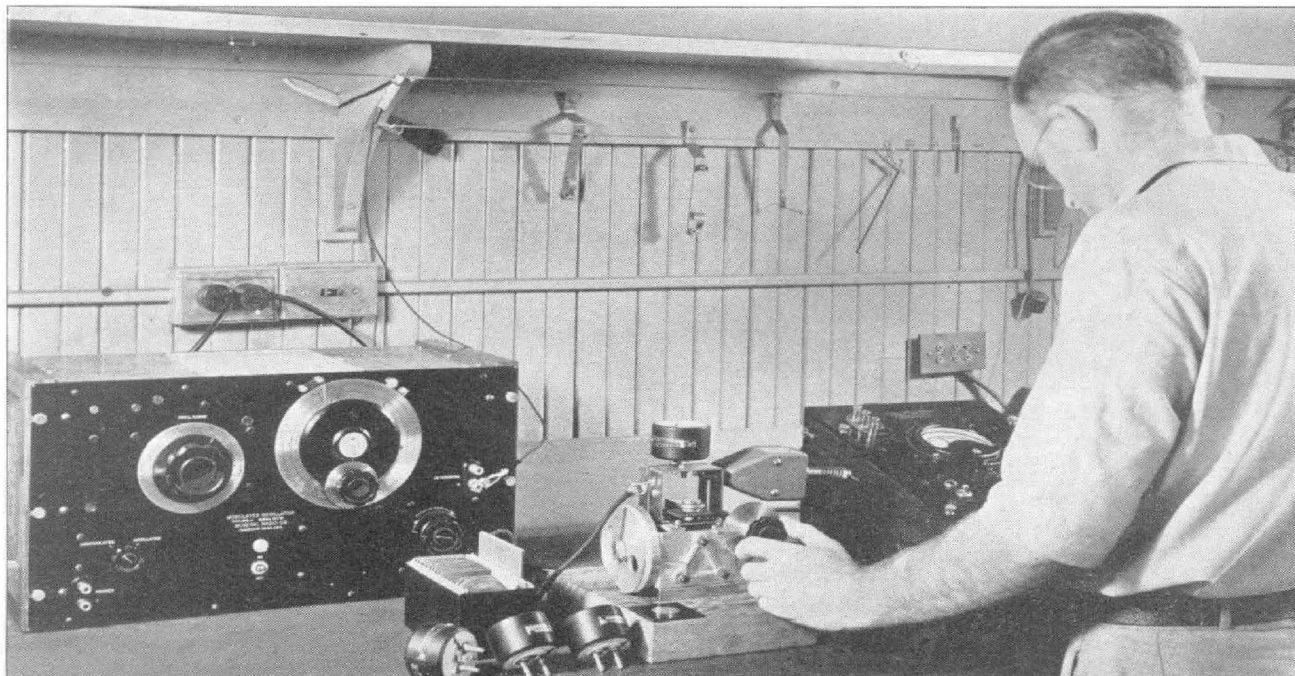TYPE 700-A Wide-Range Beat-Frequency Oscillator[1]

TYPE 684-A Modulated Oscillator[2]

TYPE 516-C Radio-Frequency Bridge[3]

[1] General Radio *Experimenter*, Vol. XIII, No. 8, p. 1, Jan., 1939.
[2] General Radio *Experimenter*, Vol. XII, No. 6, p. 1, Nov., 1937.
[3] General Radio *Experimenter*, Vol. VIII, No. 7, p. 1, Dec., 1933; Vol. XII, No. 9, p. 5, Feb., 1938; *Communications*, Vol. 19, No. 6, p. 5, June, 1939, and Vol. 19, No. 7, p. 5, July, 1939.

FIGURE 1. Laboratory set-up for measurements of power factor of dielectric samples.

Type 722-N Precision Condenser[4]

Type 500 Resistor

Type 663 Resistor[5]

Type 505 Condenser[6]

Type 670 Compensated Decade Resistor[7]

Type 726-A Vacuum-Tube Voltmeter[8]

Type 493 Thermocouples[9]

Type 755-A Condenser[10]

The need for more accurate measurements at higher frequencies has not only created a demand for better circuit elements but also for better techniques. In particular, the increase in commercial measurements has led to a search for simple and reliable methods and circuits involving, wherever possible, standard parts.

Resonance methods,[11] in general, have been found most suitable for measurements at very high frequencies because of their simplicity, flexibility, and accuracy, and because of the relatively low cost of the necessary components. A practical set-up recently used in the General Radio Company laboratories for measurements of the power factors of small dielectric samples illustrates the ready adaptability of General Radio high-frequency elements to this type of measurement.

[4] General Radio *Experimenter*, Vol. XIII, Nos. 5/6, p. 1, Oct./Nov., 1938.
[5] General Radio *Experimenter*, Vol. XIII, No. 8, p. 6, Jan., 1939.
[6] General Radio *Experimenter*, Vol. VII, No. 8, p. 1, Jan., 1933; Vol. XII, No. 11, p. 4, April, 1938.
[7] General Radio *Experimenter*, Vol. VIII, No. 10, p. 6, March, 1934.
[8] General Radio *Experimenter*, Vol. XI, No. 12, p. 1, May, 1937; Vol. XIII, Nos. 3/4, p. 1, Aug./Sept., 1938.
[9] General Radio *Experimenter*, Vol. XIII, No. 10, p. 5, March, 1939.
[10] General Radio *Experimenter*, Vol. XIV, No. 3, p. 1, Aug., 1939.
[11] For a detailed treatment of resonance methods, see D. B. Sinclair, "Parallel-Resonance Methods for Precise Measurements of High Impedances at Radio Frequencies and a Comparison with the Ordinary Series-Resonance Methods," *Proc. I.R.E.*, Vol. 26, No. 12, p. 1466, Dec., 1938.

METHOD

The photograph of Figure 1 shows the complete set-up which comprises a Type 684-A Modulated Oscillator, a Type 755-A Condenser, and a Type 726-A Vacuum-Tube Voltmeter connected in a susceptance-variation circuit.[12] The circuit diagram is illustrated in Figure 2.

The effective conductance of a parallel-resonance circuit of this type is given rigorously by the expression

$$G_e = \frac{\omega(C - C_r)}{\sqrt{\left(\frac{V_r}{V}\right)^2 - 1}} \quad (1)$$

where V_r is the voltage occurring across the parallel-resonant circuit at resonance with the variable capacitance equal to C_r, and V is the voltage occurring across the parallel-resonant circuit at any other capacitance setting, C.[13]

The effective conductance, G_e, is easily determined by measuring the voltage at parallel resonance, V_r, and the change in capacitance, $C_1 - C_2$, between the two settings at which the voltage equals $V_r / \sqrt{2}$. For these conditions Equation (1) takes the simple form

$$G_e = \frac{\omega(C_1 - C_2)}{2} \quad (2)$$

In order to measure a dielectric sample, all that is necessary is to make two conductance measurements, one with the sample disconnected and one with it connected across the tuned circuit. The resonant capacitance must be the same in both cases, so the difference in setting of the standard condenser at resonance is equal to the capacitance, C_x, of the sample. The conductance of the sample is added directly to the effective conductance of the parallel-resonant circuit so the dif-

12, 13 Ibid.

ference between the two measured conductances is equal to the conductance, G_x, of the sample. The dissipation factor, D_x,[14] of the sample can be easily computed from the capacitance and conductance by means of the relation

$$D_x = \frac{G_x}{\omega C_x} \qquad (3)$$

CORRECTIONS AND DESCRIPTION OF PHYSICAL SET-UP

The particular feature that distinguishes high-frequency measurements, in contrast with low-frequency measurements, is not any new and strange phenomenon but simply the greater relative importance of residual parameters that are inherent in the impedance standards and in the wiring.
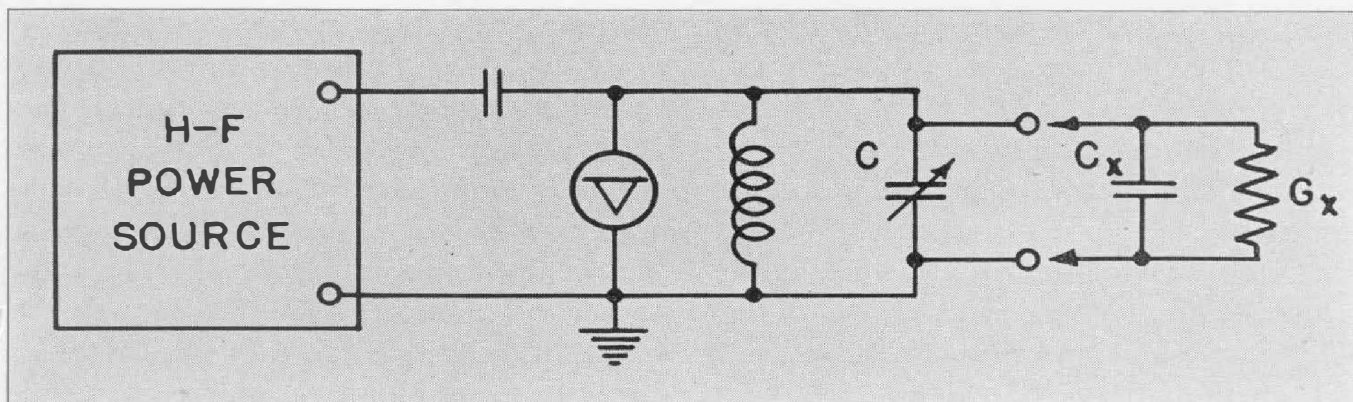
The only impedance standard used in the susceptance-variation method is the incremental admittance of the standard condenser, C, shown in Figure 2. In order to predict the limitations of this method and to compute the corrections that may be necessary, it is essential to analyze carefully the effects of residual parameters in the condenser.

In many applications, it has been found justifiable to treat a variable air condenser as a variable capacitance having no loss and no variation with frequency. At high frequencies, however, especially when used in a measuring circuit, a condenser cannot be considered so nearly ideal a circuit element. The equivalent circuit of Figure 3, to a first approximation, includes the residual parameters that cause the behavior of an actual condenser to depart from that of an ideal capacitance. The conductance, G_c, represents the loss in the dielectric structure, the resistance, R_c, represents the loss in the metallic structure, and the inductance, L_c, represents the inductance caused by magnetic flux set up by currents in the metallic structure.[15] All these residual parameters are nearly constant as a function of dial setting. At high frequencies, where skin-effect is essentially complete, the conductance, G_c, increases nearly linearly with frequency, the resistance, R_c, increases about as the square root of the frequency, and the inductance, L_c, remains constant. In the TYPE 755-A

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[14] The term power factor, rather than dissipation factor, is often used loosely to describe the ratio of resistance to reactance, or conductance to susceptance, rather than the ratio of resistance to impedance, or conductance to admittance, because the numerical value for useful dielectrics is so small that there is no significant difference between them.

[15] For a discussion of this equivalent circuit, and methods of measuring the residual parameters, see R. F. Field and D. B. Sinclair, "A Method for Determining the Residual Inductance and Resistance of a Variable Air Condenser at Radio Frequencies," *Proc. I.R.E.*, Vol. 24, No. 2, p. 255, February, 1936.

FIGURE 2. Susceptance-variation circuit used for the measurements.

Condenser, which has been especially designed for high-frequency service, the residual parameters are very small. At a frequency of 100 Mc, for instance, they are, approximately:

$$G_c = 0.0055 \text{ microhenry}$$
$$R_c = 0.05 \text{ ohm}$$
$$L_c = 100 \text{ micromhos}$$

The effects of these residual parameters depend upon the use to which the condenser is put. In the susceptance-variation circuit only admittance *differences* are used. Since the conductance, G_c, is in circuit at all times, and is constant, it drops out when taking admittance differences and causes no error.

The residual inductance, L_c, causes the effective terminal capacitance, $\hat{C}$, to differ from the static capacitance, C.

$$\hat{C} = \frac{C}{1 - \omega^2 L_c C} \quad (4)$$

Since the fractional change is a function of the static capacitance, the difference between static and effective capacitance increments is neither zero nor constant, but varies as the absolute capacitances are changed.

The residual resistance, R_c, gives rise to an effective conductance component, G_c'.

$$G_c' \simeq R_c(\omega\hat{C})^2 \quad (5)$$

Since this conductance component depends upon the capacitance setting, it does not cancel out when taking admittance differences.

Both the residual inductance, L_c, and the residual resistance, R_c, can therefore cause error in measurement. The inductance, first of all, causes error in the measurement of the circuit conductance, G_e, if static capacitance values are used in Equations (1) or (2). When the effect
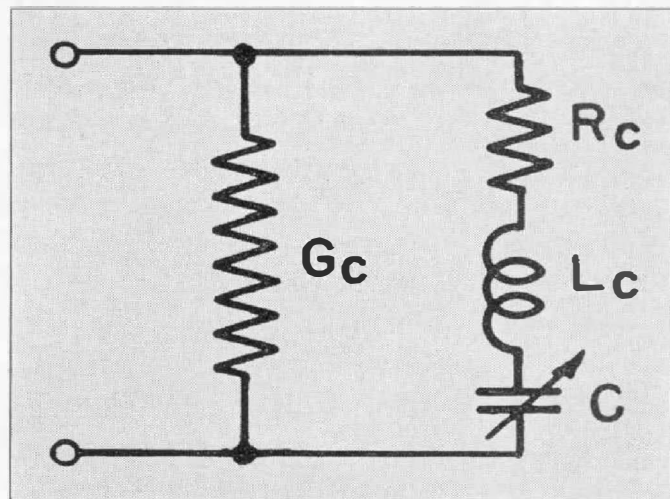

FIGURE 3. Equivalent circuit of a variable air condenser, including residual parameters.

of the residual inductance, L_c, is considered, these equations become

$$G_e = \frac{\omega(C - C_r)}{1 - \omega^2 L_c(C + C_r)} \cdot \frac{1}{\sqrt{\left(\frac{V_r}{V}\right)^2 - 1}} \quad (1a)$$

$$= \frac{1}{2} \frac{\omega(C_1 - C_2)}{1 - \omega^2 L_c(C_1 + C_2)} \quad (2a)$$

The inductance also causes error in the determination of the capacitance of the dielectric sample. If C_{r_1} and C_{r_2} are the resonant static capacitances with the sample in and out of circuit, and $\hat{C}_{r_1}$ and $\hat{C}_{r_2}$ the corresponding effective terminal capacitances,

$$C_x = \hat{C}_{r_2} - \hat{C}_{r_1} \simeq \frac{C_{r_1} - C_{r_2}}{1 - \omega^2 L_c(C_{r_1} + C_{r_2})} \quad (6)$$

The resistance causes the effective conductance of the parallel tuned circuit to change as the condenser setting is altered. While this means that, strictly speaking, the resonance curve is distorted, the effect of the conductance in determining the shape of the resonance curve is small except at settings very near the resonant value and, from a practical standpoint, the residual resistance can be considered as introducing a constant component of conductance equal to the value at the resonant capac-

itance setting. At the two resonant capacitances, C_{r_1} and C_{r_2}, the conductance components introduced by R_c are different. The difference between the two measured values of conductance, the one with the sample in circuit and the other with it out, is therefore not equal to the conductance of the sample alone but the sum of this conductance and the change in circuit conductance caused by R_c.

$$G_{e_1} - G_{e_2} = G_x + R_c\omega^2(\hat{C}_{r_1}^2 - \hat{C}_{r_2}^2)$$
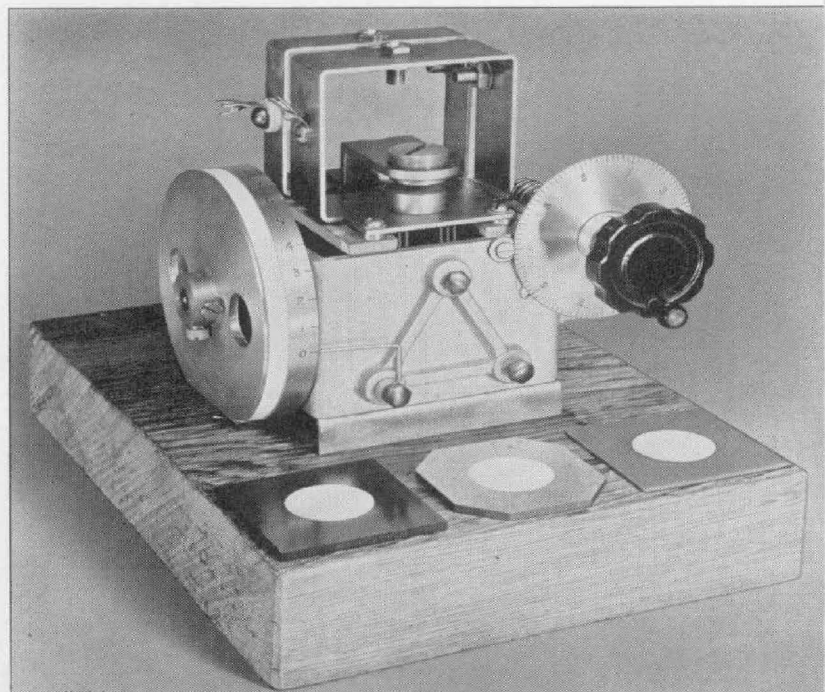$$= G_x - R_c\omega^2 C_x(\hat{C}_{r_1} + \hat{C}_{r_2}) \quad (7)$$

If no corrections for the residual inductance are used, the capacitance of the sample and the effective circuit conductance will both appear to be smaller than they actually are at very high frequencies. Since the conductance of the sample is found from the difference between the two measured conductances, it will also appear to be smaller than it actually is. If no corrections for the residual resistance are used, the value of the sample conductance will appear even smaller and may even appear to be negative, if the correction becomes sufficiently large.

Satisfactory methods of determining the magnitudes of the residual parameters and of applying the necessary corrections have been worked out and described fully.[16] The application of the corrections, however, is tedious and laborious. The use of a special type of holder for the dielectric sample, which eliminates the error caused by the residual resistance, R_c, is therefore highly desirable. The holder is shown in detail in Figure 4. It consists simply of a brass strap, which is mounted on the ungrounded terminal of the standard condenser and which extends over a brass plate screwed to the grounded terminal. To the grounded plate is attached a disc of brass, above which a threaded rod of

16 See footnotes 11 and 15.

the same diameter is tapped into the brass strap. The sample is first placed on the brass disc and the threaded rod screwed down until it touches the electrode, and a conductance measurement is made. The sample is then removed, the threaded rod is screwed down until resonance occurs at the same setting of the standard condenser, and another conductance measurement is made. Since both measurements are made at the same value of resonant capacitance in the standard condenser, the conductance components arising from the residual resistance, R_c, are equal and cancel out when taking admittance differences. The only remaining error is caused by the inductance, L_c, and this error can be corrected for by the use of Equations (1a), (2a), and (6).

The use of the special holder also avoids one of the errors caused by resid-

Figure 4. Detail of the condenser assembly. The tuning inductance and the vacuum-tube voltmeter plug into the jacks (Type 274-J) mounted on the rectangular straps. These straps are made wide to minimize the resistance and inductance of the leads to the coil, vacuum-tube voltmeter, and coupling condenser. The capacitive coupling to the power source is through the ceramic bushing shown on the left-hand side of the rear strap.

ual impedances in the lead from the sample to the standard condenser. This lead necessarily has both resistance and inductance and consequently modifies the admittance of the sample, as measured at the condenser terminals. If, however, the threaded rod is screwed down, as previously described, to give the same resonant capacitance setting with the sample in and out of circuit, the effective conductance component caused by the lead resistance will be the same both with the sample in and with it out of circuit. No correction need therefore be applied for this resistance and the only correction is for the inductance of the lead.[17]

The effect of the lead inductance, L_l, is to make the effective capacitance at the point where the leads are connected to the standard condenser, $\hat{C}_x$, greater than the true capacitance of the sample, C_x, by the factor shown in Equation (8).

$$\hat{C}_x = C_x \left(\frac{1}{1 - \omega^2 L_l C_x} \right) \qquad (8)$$

This correction can be avoided by meas-

[17] This procedure for eliminating the corrections necessary for residual resistance in the standard condenser and in the leads to the sample is adapted from a method employing special equipment described in L. Hartshorn and W. H. Ward, "The Measurement of the Permitivity and Power Factor of Dielectrics at Frequencies from 10^4 to 10^8 cycles per second," *Proc. Wireless Section I.E.E.* (London), Vol. 12, p. 6, March, 1937.

uring the capacitance of the threaded rod to the brass disc and deducing the capacitance of the sample from the amount which the rod must be turned to maintain the same resonant capacitance on the standard condenser when the sample is in and out.[18] The gap between the rod and disc, however, becomes so small when the sample is out that the holder must be very carefully made in order to obtain sufficient accuracy. The inductance, L_l, and capacitance, C_x, are both so small in the mechanical arrangement described that the correction is negligible at frequencies up to 100 Mc. The sample capacitance is therefore measured by taking the difference between the two effective resonant capacitances, first with the sample in and, second, with the sample out and the threaded rod screwed up until its bottom face is flush with the bottom of the strap.

MEASURING TECHNIQUE AND RESULTS

Curves of the capacitance and power factor of five different dielectric samples are shown in Figures 5 and 6. No corrections were made for the residual inductance, L_c, in plotting these points. The effect of inductance can be seen in
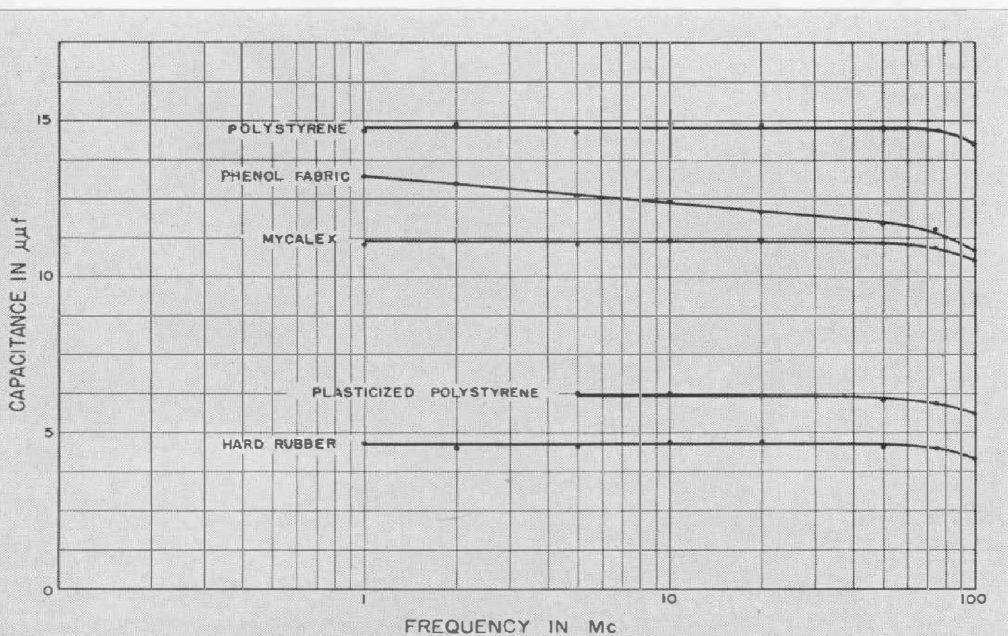
[18] See footnote 17.

FIGURE 5. Capacitance of dielectric samples as a function of frequency. The falling off of the curves between 50 and 100 Mc is caused by residual inductance in the standard condenser, for which no corrections were made. Corrections can easily be made by the application of Equation (6). The TYPE 684-A Modulated Oscillator was used at frequencies up to 50 Mc. The 75 Mc and 100 Mc points were obtained with the Peterson oscillator described in the October, 1937, issue of the *Experimenter.*

the droop of the capacitance values between 50 and 100 Mc. It is of little importance in the determinations of power factor, however, because the capacitance increments used to determine both the conductance and capacitance components fall in approximately the same capacitance region. Both are therefore affected by the inductance to roughly the same extent.

There are also corrections for stray capacitances that should be made in order to obtain results of maximum accuracy.[19] These errors are independent of frequency and arise because of the mechanical construction of the holder. The chief error occurs because the stray capacitance of the strap to ground is not the same when the sample is in and when it is out of the holder. The capacitance difference, as read on the standard condenser, is therefore not exactly equal to the capacitance of the sample. Corrections for the stray capacitance were determined from measurements

[19]Fringing corrections are not considered in this article, as they do not primarily bear upon the high-frequency measurement problem. The electrodes used on the samples were made of lead foil and were circular in shape. The diameters of the electrodes and the dimensions of the samples were not the same for all samples.

made at low frequencies where the inductance of relatively long leads was not important. Despite the dissimilarity of the samples, these corrections were found to be fairly uniform, ranging from about 0.8 $\mu\mu$f to 1.3 $\mu\mu$f. A small error in conductance also occurs because some of the stray electrostatic field from the strap passes through the sample. The error arising from this source, however, is small because the series air gaps are relatively large.
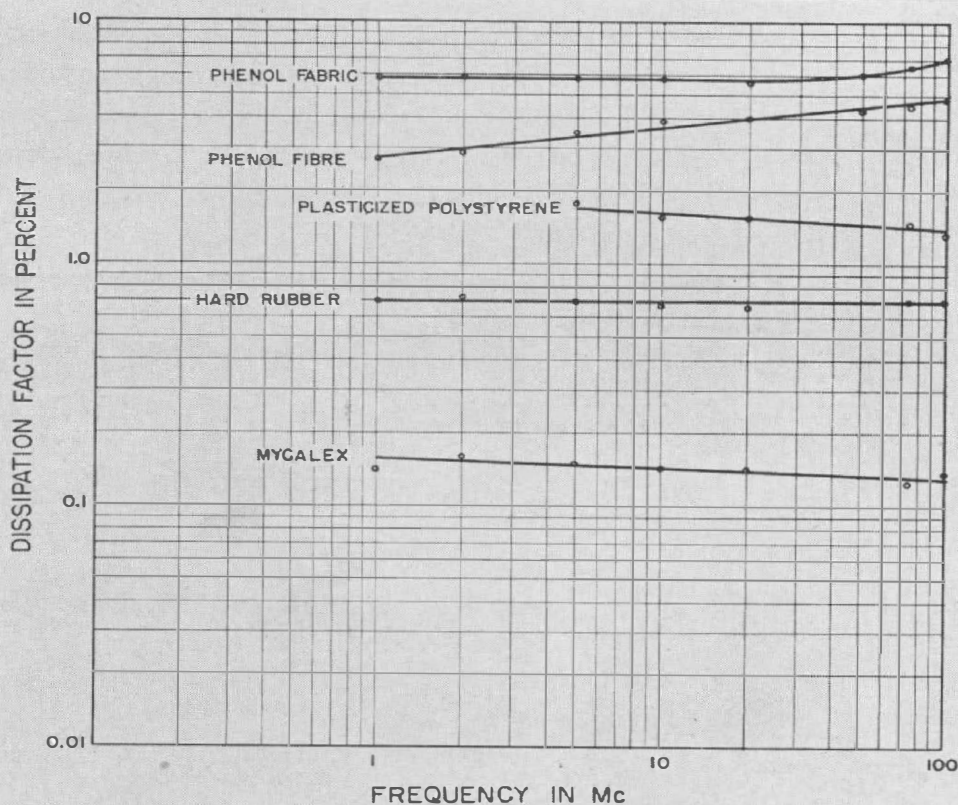
In order to obtain maximum rapidity, the following measurement routine is convenient:

(1) With the sample in the holder, determine the resonant voltage, V_{r_1}, the resonant capacitance, C_{r_1}, and the two capacitance values, C_1 and C_2, at which the voltage is equal to $V_{r_1}/\sqrt{2}$.

(2) With the sample out of the holder, screw down the threaded rod until the resonant point occurs at approximately the same capacitance setting and read the new resonant voltage, V_{r_2}.

(3) With the sample out of the holder, screw up the threaded rod until its lower

FIGURE 6. Power factor of dielectric samples as a function of frequency. No corrections for residual inductance were necessary in these measurements. Measurements were made of the power factor of the polystyrene sample, the capacitance of which is plotted in Figure 5, but the observations were inconsistent because of the extremely low power factor of this material (less than 0.03%).

face is flush with the bottom of the strap and read the resonant capacitance C_{r_2}.

The conductance, G_{e_1}, of the parallel-tuned circuit, with the sample in the holder, is found from Equation (2) or (2a). Since the resonant voltage is inversely proportional to the circuit conductance, the conductance of the sample, G_x, is easily found from the following equation:

$$G_x = G_{e_1} - G_{e_2}$$

$$= G_{e_1}\left(1 - \frac{G_{e_2}}{G_{e_1}}\right) = G_{e_1}\left(1 - \frac{V_{r_1}}{V_{r_2}}\right)$$

$$= G_{e_1}\left(\frac{V_{r_2} - V_{r_1}}{V_{r_2}}\right) \tag{9}$$

The apparent capacitance of the sample is found from Equation (6), and the true capacitance, C_x, found by adding the correction for stray capacitance to the apparent value. The dissipation factor is found from Equation (3).

— D. B. SINCLAIR

MISCELLANY

● **RECENT VISITORS** to the General Radio plant and laboratories include Mr. G. H. Marchal of the University of Brussels; Messrs. D. Mordossov, A. Vysselski, A. Yurinov, V. Tzvetkov, and P. Kuynetzov of the Glavesprom Commission of the U. S. S. R.; and Mr. R. S. Baldwin of the Bureau of Engineering, U. S. Navy Department.

● **A FEW ERRORS** appeared in the detailed specifications for TYPE 755-A Condenser, published last month. The gear ratio is 30 to 1, not 15 to 1 as stated. Mounting holes are tapped with a 10-32 thread, not 8-32. Measurements on production units indicate a minimum direct capacitance of less than 5 $\mu\mu$f. Rotor-to-ground capacitance is about 7 $\mu\mu$f, stator-to-ground, about 6 $\mu\mu$f.

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